

# Quantum Creation of a Universe in Albrecht-Magueijo-Barrow model

A.V. Yurov\*, V.A. Yurov\*\*

\*Department of Theoretical Physics, Russian State University of Immanuel Kant,  
Aleksandra Nevskogo 14, Kaliningrad 236041, Russia; e-mail [artyom\\_yurov@mail.ru](mailto:artyom_yurov@mail.ru)

\*\*Department of Mathematics, University of Missouri,  
Columbia 65201, US; e-mail [valerian@math.missouri.edu](mailto:valerian@math.missouri.edu)

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## Abstract

In quantum cosmology the closed universe can spontaneously nucleate out of the state with no classical space and time. The semiclassical tunneling nucleation probability can be estimated as  $P \sim \exp(-\alpha^2/\Lambda)$  where  $\alpha = \text{const}$  and  $\Lambda$  is the cosmological constant.

In classical cosmology with varying speed of light  $c(t)$  it is possible to solve the horizon problem, the flatness problem and the  $\Lambda$ -problem if  $c = sa^n$  with  $s = \text{const}$  and  $n < -2$ . We show that in VSL quantum cosmology with  $n < -2$  the semiclassical tunneling nucleation probability is  $P \sim \exp(-\beta^2 \Lambda^k)$  with  $\beta = \text{const}$  and  $k > 0$ . Thus, the semiclassical tunneling nucleation probability in VSL quantum cosmology is very different from that in quantum cosmology with  $c = \text{const}$ . In particular, it can be strongly suppressed for large values of  $\Lambda$ . In addition, we propose two instantons that describe the nucleation of closed universes in VSL models. These solutions are akin to the Hawking-Turok instanton in sense of  $O(4)$  invariance but, unlike to it, are both non-singular. Moreover, using those solutions we can obtain the probability of nucleation which is suppressed for large value of  $\Lambda$  too.

## 1 Introduction

One of the major requests concerning the quantum cosmology is a reasonable specification of initial conditions in early universe, that is in close vicinity of the Big Bang. The three wave functions, describing the quantum cosmology has been proposed so far: the Hartle-Hawking's [1], the Linde's [2], and the so-called tunneling wave function [3]. In the last case the universe can tunnel through the potential barrier to the regime of unbounded expansion. Following Vilenkin [4] lets consider the closed ( $k = +1$ ) universe filled with radiation ( $w = 1/3$ ) and  $\Lambda$ -term ( $w = -1$ ). One of the Einstein's equations can be written as a law of a conservation of the (mechanical) energy:  $P^2 + U(a) = E$ , where  $P = -a\dot{a}$ ,  $a(t)$  is the scale factor, the "energy"  $E = \text{const}$  and the potential

$$U(a) = c^2 a^2 \left( 1 - \frac{\Lambda a^2}{3} \right),$$

where  $c$  is the speed of light. The maximum of the potential  $U(a)$  is located at  $a_e = \sqrt{3/2\Lambda}$  where  $U(a_e) = 3c^2/(4\Lambda)$ . The tunneling probability in WKB approximation can be estimated as

$$P \sim \exp \left( -\frac{2c^2}{8\pi G \hbar} \int_{a'_i}^{a_i} da \sqrt{U(a) - E} \right), \quad (1)$$

where  $a'_i < a_i$  are two turning points. The universe can start from  $a = 0$  singularity, expand to a maximum radius  $a'_i$  and then tunnel through the potential barrier to the regime of unbounded expansion with the semiclassical tunneling probability (1). Choosing  $E = 0$  one gets  $a'_i = 0$  and  $a_i = \sqrt{3/\Lambda}$ . The integral in (1) can be calculated. The result can be written as

$$P \sim \exp \left( -\frac{2c^3}{8\pi G \hbar \Lambda} \right). \quad (2)$$

For the probability to be of reasonable value, for example  $P = 1/e \sim 0.368$ , one has to put  $\Lambda \sim 0.3 \times 10^{65} \text{ cm}^{-2}$  (see (2)). In other words, the  $\Lambda$ -term must be large. On the other side, the universe once nucleated immediately begins a de Sitter inflationary expansion. Therefore the tunneling wave function results in inflation. And the  $\Lambda$ -term problem, which arises in this approach is usually being gotten rid of via the anthropic principle. In this case we have two Lorentzian regions ( $0 < a < a'_i$ ,  $a > a_i$ ) and one Euclidean region ( $a'_i < a < a_i$ ). The second turning point  $a = a_i$  corresponds to the beginning of our universe. If  $\Lambda = 0$  then  $U(a)$  has the form of **parabola** and we get only one Lorentzian region. In this case, the universe can start at  $a = 0$ , expand to a maximum radius and recollapse. If  $E \rightarrow 0$ , the single Lorentzian region contracts to a single point, which lies in agreement with the tunnelling nucleation probability:  $P \rightarrow 0$  as  $\Lambda \rightarrow 0$ . However, as we'll show, in quantum cosmological VSL models the situation can be opposite, viz: the probability to find the finite universe short after it's tunneling through the potential barrier is  $P \sim \exp(-\beta(n)\Lambda^{\alpha(n)})$  with  $\alpha(n) > 0$  and  $\beta(n) > 0$  when  $n < -2$  or for  $-1 < n < -2/3$ . After the tunneling one gets the finite universe with "initial" value of scale factor  $a_i \sim \Lambda^{-1/2}$ , so the probability to find the universe with large value of  $\Lambda$  and small value of  $a_i$  is strongly suppressed. The reason for this lies in the behavior of potential  $U(a)$ , which, for the case  $\Lambda \rightarrow 0$ , transforms into the **hyperbola**, located under the abscissa axis. As a result, such a universe can at  $a \sim 0$  start the regime of unbounded expansion. Therefore, we get the single Lorentzian region that doesn't contract to a point at  $E \rightarrow 0$ .

This new property of VSL quantum cosmology will be discussed in next Section but new question arouse: the geometric interpretation of the quantum creation of a Universe with varying speed of light. We know that universe can be spontaneously created from nothing (when  $c = \text{const}$ ) and this process can be described with the aid of the instantons solutions possessing  $O(5)$  (if  $V(\phi)$  has a stationary point at some nonzero value  $\phi = \phi_0 = \text{const}$ ) or  $O(4)$  (as Hawking-Turok instanton [5]) invariance. So, what can be said about instantons in the VSL models?

The whole plan of the paper looks as follows: in the next Section we'll consider the simplest VSL model: model of Albrecht-Magueijo-Barrow. Then we show that in framework of tunneling approach to quantum cosmology with VSL the semiclassical tunneling nucleation probability can be estimated as  $P \sim \exp(-\beta^2 \Lambda^k)$  with  $\beta = \text{const}$  and  $k > 0$ . All corresponding calculations will be done for the case of the universe filled with radiation ( $w = 1/3$ ) and vacuum energy. In the Section 3 we'll propose the **non-singular** instanton solutions possessing only  $O(4)$  invariance (so the Euclidean region is a deformed four sphere). These solutions can in fact lead to inflation after the analytic continuation into the Lorentzian region. We will discuss these results in Sec. 4.

## 2 Albrecht-Magueijo-Barrow VSL model

Lets start with the Friedmann and Raychaudhuri system of equations with  $k = +1$  (we assume the  $G=\text{const}$ ):

$$\begin{aligned} \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right) + \frac{\Lambda c^2}{3}, & \left( \frac{\dot{a}}{a} \right)^2 &= \frac{8\pi G\rho}{3} - k \left( \frac{c}{a} \right)^2 + \frac{\Lambda c^2}{3}, \\ c &= c_0 \left( \frac{a}{a_0} \right)^n = sa^n, & p &= wc^2\rho, \end{aligned} \quad (3)$$

where  $a = a(t)$  is the expansion scale factor of the Friedmann metric,  $p$  is the fluid pressure,  $\rho$  is the fluid density,  $k$  is the curvature parameter (we put  $k = +1$ ),  $\Lambda$  is the cosmological constant,  $c_0$  is some fixed value of speed of light which corresponds to a fixed value of scale factor  $a_0$ .

Using (3) one gets

$$\dot{\rho} = -\frac{3\dot{a}}{a} \left( \rho + \frac{p}{c^2} \right) + \frac{\dot{c}c(3 - a^2\Lambda)}{4\pi Ga^2}. \quad (4)$$

Choosing  $w = 1/3$  one can solve (4) to receive

$$\rho = \frac{M}{a^4} + \frac{3s^2na^{2(n-1)}}{8\pi G(n+1)} - \frac{s^2n\Lambda a^{2n}}{8\pi G(n+2)}, \quad (5)$$

where  $M > 0$  is a constant characterizing the amount of radiation. It is clear from the (5) that the flatness problem can be solved in a radiation-dominated early universe by an interval of VSL evolution if  $n < -1$ , whereas the problem of  $\Lambda$ -term can be solved only if  $n < -2$ . The evolution equation for the scale factor  $a$  (the second equation in system (3)) can be written as

$$p^2 + U(a) = E, \quad (6)$$

where  $p = -a\dot{a}$  is the momentum conjugate to  $a$ ,  $E = 8\pi GM/3$  and

$$U(a) = \frac{s^2a^{2n+2}}{n+1} - \frac{2s^2\Lambda a^{2n+4}}{3(n+2)}. \quad (7)$$

The potential (7) has one maximum at  $a = a_e = \sqrt{3/(2\Lambda)}$  such that

$$U_e \equiv U(a_e) = \frac{s^23^{n+1}}{2^{n+1}\Lambda^{n+1}(n+1)(n+2)}, \quad (8)$$

so  $U_e > 0$  if (i)  $n < -2$  or (ii)  $n > -1$ . The first case allows us to solve the flatness and "Lambda" problems. Another benefit of the model is a finite time region with accelerated expansion.

### 2.1 The semiclassical tunneling probability in VSL models with $n < -2$ : the case $E \ll U_e$

One can choose  $n = -2 - m$  with  $m > 0$ . Such a substitution gives us the potential (7) in the form

$$U_m(a) = \frac{s^2}{a^{2(m+1)}} \left( \frac{2\Lambda a^2}{3m} - \frac{1}{m+1} \right). \quad (9)$$

Since (6) is similar to equation of movement of the particle of energy  $E$  in the potential (9), the universe in quantum cosmology can start at  $a \sim 0$ , expand to the maximum radius  $a'_i$  and then tunnel through the potential barrier to the regime of unbounded expansion with "initial" value  $a = a_i$ . The semiclassical tunneling probability can be estimated as

$$P \sim \exp \left( -2 \int_{a'_i}^{a_i} |\tilde{p}(a)| da \right), \quad (10)$$

with

$$|\tilde{p}(a)| = \frac{c^2(t)}{8\pi G \hbar} |p(a)|, \quad |p(a)| = \sqrt{U_m(a) - E},$$

where  $E \leq U_e$ . It is convenient to write  $E = U_e \sin^2 \theta$ , with  $0 < \theta < \pi/2$ .

For the case  $E \ll U_e$  one can choose

$$a'_i \sim a_1 = \sqrt{\frac{3m}{2(m+1)\Lambda}}, \quad a_i \sim \sqrt{\frac{3}{2\Lambda}} \left( \frac{\sqrt{m+1}}{\sin \theta} \right)^{1/m}, \quad (11)$$

and evaluate the integral (10) as

$$P \sim \exp \left( \frac{-s^3 \Lambda^{2+3m/2} I_m(\theta)}{4\pi G \hbar} \right), \quad (12)$$

where

$$I_m(\theta) = \int_{z'_i(\theta)}^{z_i(\theta)} dz z^{-5-3m} \sqrt{\frac{2z^2}{3m} - \frac{1}{m+1}}, \quad (13)$$

with

$$z'_i(\theta) = \sqrt{\frac{3m}{2(m+1)}}, \quad z_i(\theta) = \sqrt{1.5} \left( \frac{(m+1)^{1/2}}{\sin \theta} \right)^{1/m}.$$

One can show that  $I_m(\theta) > 0$  at  $0 < \theta \ll 1$ . Thus, it is easy to see from (12) that the semiclassical tunneling probability  $P \rightarrow 0$  for large values of  $\Lambda > 0$  and  $P \rightarrow 1$  at  $\Lambda \rightarrow 0$ .

Note, that the case  $c=\text{const}$  can be obtained by substitution  $m = -2$  into the (12). Not surprisingly, this case will get us the well known result  $P \sim \exp(-1/\Lambda)$  (see [4]).

## 2.2 The semiclassical tunneling probability with $n < -2$ and $n > -1$

In the case of general position the semiclassical tunneling probability with  $n = -2 - m$  has the form

$$P_m \sim \exp \left( -\frac{s^3 \Lambda^{(3m+4)/2}}{4\pi G \hbar 3^{(m+1)/2} \sqrt{m(m+1)}} \int_{z'_i}^{z_i} \frac{dz \sqrt{F_m(z, \theta)}}{z^{3m+5}} \right), \quad (14)$$

where

$$F_m(z, \theta) = -2^{m+1} \sin^2 \theta z^{2(m+1)} + 2 \times 3^m (m+1) z^2 - m 3^{m+1}, \quad (15)$$

$z$  is dimensionless quantity and  $z'_i, z_i$  are the turning points, i.e. two real positive solutions of the equation  $F_m(z, \theta) = 0$  for the given  $\theta$  ( $F_m(z, \theta) = 0$  does have two such solutions at  $0 < \theta < \pi/2$ ).

If  $m$  is the natural number then the expression (14) has a more simple form. For example

$$P_1 \sim \exp \left( -\frac{s^3 \Lambda^{7/2} \sin \theta}{6\pi G \hbar \sqrt{2}} \int_{z'_i}^{z_i} \frac{dz}{z^8} \sqrt{(z^2 - z'^2)(z_i^2 - z^2)} \right),$$

with

$$z'_i = \frac{\sqrt{3}}{2 \cos(\theta/2)}, \quad z'_i = \frac{\sqrt{3}}{2 \sin(\theta/2)}.$$

Similarly,  $P \sim \exp(-S)$ , with

$$S = \frac{s^3 \Lambda^5 \sin \theta}{18\pi G \hbar} \int_{z'_i}^{z_i} \frac{dz}{z^{11}} \sqrt{(z^2 + z_1^2)(z^2 - z'^2)(z_i^2 - z^2)},$$

where

$$z_1 = \sqrt{\frac{3}{\sin \theta} \cos \left( \frac{\theta}{3} - \frac{\pi}{6} \right)}, \quad z'_i = \sqrt{\frac{3}{\sin \theta} \sin \frac{\theta}{3}}, \quad z_i = \sqrt{\frac{3}{\sin \theta} \cos \left( \frac{\theta}{3} + \frac{\pi}{6} \right)},$$

and so on.

Therefore the probability to obtain (via quantum tunneling through the potential barrier) the universe in the regime of unbounded expansion is strongly suppressed for large values of  $\Lambda$  and small values of the initial scale factor  $a_i = \sqrt{3}/(2 \sin(\theta/2) \sqrt{\Lambda})$ . In other words, overwhelming majority of universes born via the quantum tunneling through the potential barrier (7) have large initial scale factors and small values of  $\Lambda$ .

Now, let us consider the case (ii), when  $n > -1$ . The "quantum potential" has the form

$$U(a) = s^2 a^{2m} \left( \frac{1}{m} - \frac{2\Lambda a^2}{3(m+1)} \right), \quad (16)$$

where  $m = n + 1 > 0$ . The points of intersection with the abscissa axis  $a$  are  $a_0 = 0$  and  $a_1 = \sqrt{3(m+1)/2\Lambda m}$ . Choosing  $E = 0$  in equation (6) and substituting (16) into the (10) we get

$$P \sim \exp \left( -\frac{s^3 \Lambda^{(1-3m)/2}}{4\pi G \hbar} \int_0^{z_1} z^{2m-2} \sqrt{\frac{1}{m} - \frac{2z^2}{3(m+1)}} dz \right),$$

with  $z_1 = \sqrt{3(m+1)/2m}$  (The starting value  $z = 0$  means that the Universe tunneled from "nothing" to a closed universe of a finite radius  $a_1 = z_1/\sqrt{\Lambda}$ ). Thus, we have the same effect as if  $0 < m < 1/3$ .

### 2.3 Peculiar cases with $n = -1$ and $n = -2$

At last, lets consider the cases of  $n = -1$  and  $n = -2$ . The formula (14) is not valid in these cases ( $m = -1$  and  $m = 0$ ) so we shall consider these models separately.

If  $n = -1$  ( $m = -1$ ) then

$$\rho = \frac{M}{a^4} + \frac{\Lambda s^2}{8\pi G a^2} - \frac{3s^2}{4\pi G a^4} \log \frac{a}{a_*},$$

therefore

$$U(a) = s^2 \left( 2 \log \left( \frac{a}{a_*} \right) - \frac{2a^2 \Lambda}{3} + 1 \right), \quad (17)$$

where  $a_*$  is constant and  $[a_*] = \text{cm}$ . The potential (17) has one maximum at  $a = a_e = \sqrt{3/(2\Lambda)}$  such that  $U_e = U(a_e) = 2s^2 \log(a_e/a_*)$ , so if  $a_e > a_*$  then  $U_e > 0$ . We choose  $a_* = \Lambda^{-1/2}$ . This gives us  $U_e = 0.41s^2 > 0$ . For the case  $E \ll U_e$  the semiclassical tunneling nucleation probability is

$$P_{-1} \sim \exp \left( -\frac{s^3 \sqrt{\Lambda}}{4\pi G \hbar} \int_{z'_i}^{z_i} \frac{dz}{z^2} \sqrt{\log z^2 - \frac{2z^2}{3} + 1} \right) \sim \exp \left( -\frac{s^3 \sqrt{\Lambda}}{10\pi G \hbar} \right), \quad (18)$$

where the turning points are  $z'_i = 0.721$ ,  $z_i = 1.812$ . As we can see from the (18), when  $n = -1$  we receive the aforementioned effect again.

If  $n = -2$  ( $m = 0$ ) then

$$\rho = \frac{M}{a^4} + \frac{s^2 \Lambda}{2\pi G a^4} \log \left( \frac{a}{a_*} \right) + \frac{3s^2}{4\pi G a^6}.$$

We choose  $a_* = 1/(\alpha \sqrt{\Lambda})$ , where  $\alpha$  is a dimensionless quantity. Thus

$$U(a) = -s^2 \left( \frac{1}{a^2} + \frac{4\Lambda}{3} \log \left( \alpha a \sqrt{\Lambda} \right) + \frac{\Lambda}{3} \right). \quad (19)$$

The maximum of potential (19) is located at the same point  $a_e$  and

$$U_e = -\frac{s^2 \Lambda}{3} \left( 3 + \log \left( \frac{9\alpha^4}{4} \right) \right).$$

Therefore,  $U_e > 0$  if  $\alpha < 2e^{-3/4}/\sqrt{6} \sim 0.386$ . Choosing  $\alpha = 0.286$  and  $E \ll U_e$  gets us the turning points  $z'_i \sim 0.77$  and  $z_i \sim 2.391$ .

At last, the semiclassical tunneling nucleation probability is

$$P_0 \sim \exp \left( -\frac{s^3 \Lambda^2}{4\pi G \hbar} \int_{z'_i}^{z_i} \frac{dz}{z^4} \sqrt{-\frac{1}{z^2} - \frac{4}{3} \log(\alpha z) - \frac{1}{3}} \right) \sim \exp \left( -\frac{0.084 s^3 \Lambda^2}{\pi G \hbar} \right).$$

### 3 Instantons

If we are going to describe the quantum nucleation of universe we should find the instanton solutions, simply putted as a stationary points of the Euclidean action. The instantons give a dominant contribution to the Euclidean path integral, and that is the reason of our interest in them.

First at all, lets consider the  $O(4)$ -invariant Euclidean spacetime with the metric

$$ds^2 = c^2(\tau) d\tau^2 + a^2(\tau) (d\psi^2 + \sin^2 \psi d\Omega_2^2). \quad (20)$$

In the case  $c = \text{const}$  one can construct the simple instantons, which are the  $O(5)$  invariant four-spheres. Then one can introduce the scalar field  $\phi$ , whose (constant) value  $\phi = \phi_0$  is chosen as the one providing the extremum of potential  $V(\phi)$ . The scale factor will be  $a(\tau) = H^{-1} \sin H\tau$  and after the analytic continuation into the Lorentzian region one will get the de Sitter space or inflation. Many other examples of non-singular and singular instantons were presented in [6]

Now, lets consider the VSL model with scalar field. The corresponding Euclidean equations are:

$$\begin{aligned} \phi'' + 3\frac{a'}{a}\phi' &= \frac{c^2 V'}{\phi'} + \frac{c^5 c' (\Lambda a^2 - 3)}{4\pi G a^2 \phi'} + \frac{2\phi' c'}{c} - \frac{2cVc'}{\phi'}, \\ \left( \frac{a'}{a} \right)^2 &= \frac{8\pi G}{3c^4} \left( \frac{\phi'^2}{2} - c^2 V \right) + \frac{c^2}{a^2} - \frac{\Lambda c^2}{3}, \end{aligned} \quad (21)$$

where primes denote derivatives with respect to  $\tau$ .

At the next step we represent the potential  $V$  in factorized form

$$V = F(a)U(\phi). \quad (22)$$

Indeed, lets for example consider the power-low potential  $\sim \phi^k$ . If the coupling  $\lambda$  is dimensionless one then we get

$$V \sim \frac{\lambda}{\hbar} G^{k/2-2} c^{7-2k} \phi^k.$$

Since  $c = sa^n$  then in the simplest case we come to (22).

Let  $\phi = \phi_0 = \text{const}$  be solution of the (21). (Note, that we don't require  $\phi_0$  to be an extremum of potential.) Using the first equation of system (21) and (22) we get the equation for the  $F(a)$ ,

$$\frac{dF(a)}{da} - \frac{2n}{a} F(a) = \frac{3ns^4}{4\pi GU_0} a^{4n-3} - \frac{ns^4\Lambda}{4\pi GU_0} a^{4n-1}, \quad (23)$$

where  $U_0 = U(\phi_0) = \text{const}$ . The integration of the (23) results in

$$F(a) = a^{2n} \left( C - \frac{3ns^4}{8\pi G(1-n)U_0} a^{2(n-1)} - \frac{s^4\Lambda}{8\pi GU_0} a^{2n} \right), \quad (24)$$

where  $C$  is the constant of integration and by assumption  $n \neq -1$  and  $n \neq 0$ . Substitution of (24) into the second equation of the system (21) transforms it into the the model of nonlinear oscillator, integration of which result in

$$\frac{a'^2}{2} + u(a) = 0, \quad (25)$$

where

$$u(a) = \frac{\omega^2 a^2}{2} - \frac{s^2 a^{2n}}{2(1-n)}, \quad (26)$$

with  $\omega^2 = 8\pi GU_0 C / (3s^2)$  and with the choice  $C > 0$  made. We can see that for  $c = \text{const}$  (i.e.  $n = 0$ ) (26) turns out to be an equation of the harmonic oscillator and we come to the well-known  $O(5)$  solution (but in this case  $\phi_0$  must be the stationary point of  $V$ ).

Equation (25) naturally describes the "movement of a classical particle" with zero-point energy in mechanical potential (26). Depending on value of  $n$  this potential can take one of four distinct forms (excluding the well-known classical case  $n = 0$ , which lies beyond the scoop of this article).

**Case 1:**  $n < 0$ . Here we have one Euclidean ( $0 \leq a \leq a_1$ ) and one Lorentzian ( $a > a_1$ ) regions where

$$a_1 = \left( \frac{s}{\omega \sqrt{1-n}} \right)^{1/(1-n)}. \quad (27)$$

On the bound between Euclidean and Lorentzian regions ( $a = a_1$ ) we have  $a' = 0$ .

This mechanical potential is unbounded from below at  $a \rightarrow 0$ . With this in mind, we'll have to ascertain that the Euclidean action for our solution will stay finite. The gravitation action has the form

$$S_{\text{grav}} = - \int d^4x \frac{c^3}{8\pi G} \sqrt{g} R.$$

We are using the dimensionless variables  $x^0 = c_0 \tau / a_0$ ,  $x^1 = \psi$  and so on. Calculating  $R$  we get

$$R = \frac{6}{c_0^2 a^2} \left[ c_0^2 - \left( \frac{a_0}{a} \right)^{2n} ((1-n)a'^2 + aa'') \right], \quad (28)$$

so we do have the potential divergence at  $a = 0$ . Multiplying (28) on the  $\sqrt{g}$  and  $c^3$  and using the equation of motion we get the expression:

$$R\sqrt{g}c^3 \sim 6c_0 \left( (2-n)\omega^2 \frac{a^{2n+3}}{a_0^{2n-1}} - \frac{nc_0^2}{1-n} \frac{a^{4n+1}}{a_0^{4n-1}} \right), \quad (29)$$

where the most dangerous multiplier factor is  $a^{1+4n}$ . But if  $-1/4 \leq n < 0$  then the Euclidean action becomes finite and therefore, we end up with the legitimate gravitation instanton. In a similar manner, using (22) and (24) we get for the scalar field (in dimensionless  $x^\mu$ ):

$$\sqrt{g}V_0 \sim \frac{c_0 a_0^{1-3n}}{8\pi G} (3\omega^2 a^{3(1+n)} + \frac{3nc_0^2 a^{1+5n}}{(n-1)a_0^{2n}} - \frac{\Lambda c_0^2 a^{3+5n}}{a_0^{2n}})$$

therefore the instanton exists for  $n > -1/5$ . This requirement is stronger than the one for the gravitation instanton where  $n > -1/4$  (see (29)).

**Case 2.**  $0 < n < 1$ . Here the potential  $u(a)$  suffers no singularity at  $a = 0$ , but  $u(0) = 0$ . Also this potential has a minimum at

$$a_0 = \left( \frac{s}{\omega} \sqrt{\frac{n}{1-n}} \right)^{1/(1-n)},$$

and is equal to zero at (27), hence, once again creating one Euclidean and one Lorentzian regions, separated by (27).

**Case 3.**  $n = 1$ . This case is somehow special, since for such  $n$  the solution of (23) shall be

$$F(a) = a^{2n} \left( C - \frac{3s^4}{4\pi GU_0} \ln a - \frac{s^4 \Lambda}{8\pi GU_0} a^2 \right),$$

instead of (24), and hence, the equation of (26) shall be substituted by

$$u(a) = a^2 \left( \frac{\omega^2}{2} - s^2 \ln a \right). \quad (30)$$

It is easy to see that this function has two zeros (at  $a_1 = 0$  and  $a_2 = \exp(\frac{\omega^2}{2s^2})$ ), is strictly positive on interval  $(a_1, a_2)$  and strictly negative outside of it. Therefore, this case doesn't allow an instanton.

**Case 4.**  $n > 1$ . The potential  $u(a)$  is strictly positive. The instanton doesn't exist either.

Both of a newly founded solutions possess only  $O(4)$  invariance just like Hawking-Turok instanton (so the Euclidean region is a deformed four sphere) but, unlike to it, they are all non-singular. Note that if the value  $a$  is sufficiently large then one can neglect the second term in (26) (after the analytic continuation into the Lorentzian region) therefore, as in the case of the usual  $O(5)$  instanton, one can get the de Sitter universe, i.e. the inflation.

The equation (25) has no terms with  $\Lambda$ . In other words, the scale factor  $a(\tau)$  doesn't depend on the value  $\Lambda$  (although being dependant on the  $U_0$ ). Therefore, the full Euclidean action  $S_E = S_{\text{grav}} + S_{\text{field}}$  has the form,

$$S_E = S_0 - \Lambda S_1,$$

where  $S_0$  and  $S_1$  are both independent of the  $\Lambda$ . Returning to what has been said in Introduction, there exist three common ways to describe the quantum cosmology: the Hartle-Hawking wave function  $\exp(-S_E/\hbar)$ , the Linde wave function  $\exp(+S_E/\hbar)$  and the tunneling wave function. In

the second Section we have been working with the tunneling wave function. In case of instantons situation becomes slightly different. If  $S_1 > 0$  then (as a first, tree semiclassical approximation) we should choose the Linde wave function, whereas for the case  $S_1 < 0$  the Hartle-Hawking wave function seems more naturally.

In conclusion, we note that another choice of  $C$  ( $C < 0$  and  $C = 0$ ) eliminates all possible instantons.

## 4 Discussion

VSL models contain both some of the promising positive features [7] and some shortcomings and unusual (unphysical?) features as well [8]. But, as we have shown, application of the VSL principle to the quantum cosmology indeed results in amazing previously unexpected observations. The first observation is that the semiclassical tunneling nucleation probability in VSL quantum cosmology is quit different from the one in quantum cosmology with  $c=\text{const}$ . In the first case this probability can be strongly suppressed for large values of  $\Lambda$  whereas in the second case it is strongly suppressed for small values of  $\Lambda$ . This is interesting, although we still can't say that VSL quantum cosmology definitely results in solution of the  $\Lambda$ -mystery. The problem here is the validity the WKB wave function. And what is more, throughout the calculations we have been omitting all preexponential factors (or one loop quantum correction) which can be essential ones near the turning points. Another troublesome question is the effective potentials in VSL models, being unbounded from below at  $a \rightarrow 0$ . The naive way to solve this problem is to use the Heisenberg uncertainty relation to find those potentials with the ground state. However, this is just a crude estimation. To describe the quantum nucleation of universe we have to find the instanton solution which, being a stationary point of the Euclidean action, gives the dominant contribution to the Euclidean path integral. As we have seen, such solutions indeed exist in VSL models. Those instantons are  $O(4)$  invariant, are non-singular, and provide an inflation as well. They describe the quantum nucleation of universe from "nothing" and, what is more, upon usage of these solutions we can obtain the probability of a nucleation which is suppressed for large value of  $\Lambda$  using either Linde or Hartle-Hawking wave function.

Note, that we can weaken the condition  $n > -1/5$  to obtain a singular instanton suffering the integrable singularity (i.e. such that the instanton action will be finite) in the way of the Hawking-Turok instanton. However, there exist some arguments [9], that such singularities, even being integrable, still lead to serious problems with solutions.

In conclusion, we note that obtained instantons both have a free parameter ( $\omega^2$ ) so we are free to use the anthropic approach to find the most probable values of  $\Lambda$  too.

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